Cognitive Neurodynamics
https://doi.org/10.1007/s11571-020-09574-w

RESEARCH ARTICLE

Removing uncertainty in neural networks

Arturo Tozzi[1] · **James F. Peters**[2,3]

Received: 23 August 2019 / Revised: 24 January 2020 / Accepted: 19 February 2020

Abstract
Neuroscientists draw lines of separation among structures and functions that they judge different, arbitrarily excluding or including issues in our description, to achieve positive demarcations that permits a pragmatic treatment of the nervous activity based on regularity and uniformity. However, uncertainty due to disconnectedness, lack of information and absence of objects' sharp boundaries is a troubling issue that prevents these scientists to select the required proper sets/subsets during their experimental assessment of natural and artificial neural networks. Starting from the detection of metamorphoses of shapes inside a Euclidean manifold, we propose a technique to detect the topological changes that occur during their reciprocal interactions and shape morphing. This method, that allows the detection of topological holes development and disappearance, makes it possible to solve the problem of uncertainty in the assessment of countless dynamical phenomena, such as cognitive processes, protein homeostasis deterioration, fire propagation, wireless sensor networks, migration flows, and cosmic bodies analysis.

Keywords Holes · Grid · Homology · Shapes · Uncertainty

Introduction

Demarcation among objects and things is somewhat arbitrary, because our mind tends to exclude the continuity among hidden or unknown structures of the world (Fort 1919; Popkin and Maia Neto 2007; Autrecourt 1340; Tozzi and Peters 2019). In touch with set theory, observers tend to spatially and temporally split the set of the entire world in different, sometimes arbitrary, subsets. This concept seems straightforward in math, and in particular in topology, where, the Jordan theorem states that exist an "inside" and an "outside" region (Ghrist 2014; Arkhangelskii 2001; Terasawa 2004). The tremendous power of topological set and cell complex theory (Peters 2018a, b; Cooke and Finney 1967; Alexandroff 1961) has been successfully used in many applications, from math to logic, from physics to biology, from neuroscience to statistics. In a covering of a finite, bounded plane region, for example, there are three elementary cells to consider, namely, 0-cells (vertexes), 1-cells (edges) and 2-cells (filled triangles). The core of set and cell complex theory lies in the occurrence of path-connected sets or subsets on surfaces containing punctures (holes), i.e., the presence of elements which are (more or less arbitrarily) joined together and separated from the external environment. There is an analogy between usual presence of tiny pores (punctures) in physical surfaces and aberrant surface punctures (large holes) and, for example, protein homeostasis in the human body, namely, healthy neuronal homeostasis versus aberrant neuronal perturbations (Harnack et al. 2015) and cellular mechanics (translation, folding and clearance) of the proteome (Morimoto and Cuervo 2009).

A serious problem arises when we grasp that, in many applications, a system's object shapes are unknown and apparently disconnected, making it very difficult to find and recognize the proper subtending sets' very presence and/or dynamics (Déli et al. 2017; Wang and Hu 2019).

✉ Arturo Tozzi
tozziarturo@libero.it; Arturo.Tozzi@unt.edu

James F. Peters
James.peters3@umanitoba.ca

1 Center for Nonlinear Science, Department of Physics, University of North Texas, 1155 Union Circle, #311427, Denton, TX 76203-5017, USA

2 Department of Electrical and Computer Engineering, University of Manitoba, Winnpeg, MB R3T 5V6, Canada

3 Department of Mathematics, Adıyaman University, 02040 Adıyaman, Turkey

Published online: 27 February 2020

The boundaries of object shapes may be very irregular and indistinct, leading to a knowledge of the split between shape interiors and boundaries which hampers shape approximation and measurement (Tozzi and Peters 2017; Rao 2018; Sen et al. 2019). In this paper, we will start from moving objects, i.e., features which can be described by a function from time to its corresponding spatial object (Gäuting et al. 2000). Similarly, a moving region related to a moving object displays mappings from time to a region object, i.e., a feature consisting of one or several connected components called faces, each of which may or may not have holes inside it (Schneider and Behr 2006). In this paper, in order to tackle the painstaking issues related to unavailable topological spatial relationships and intrinsic uncertainty, we tackle the goal to assess the trajectories of undefined/disjointed/disconnected sets or subsets: in particular, we consider changes in topological features of two unknown spatial shapes moving inside a two-dimensional grid.

Materials and methods

Several approaches to the study of moving and region objects use procedures of data analysis arising from persistent homology, whose early roots (for connected components) trace back to SizeTheory (Verri et al. 1993; Verri and Uras 1997), later generalized to higher dimensions (Edelsbrunner et al. 2002; Edelsbrunner and Morozov 2012). Among the models which assess topological relationships, i.e., the topological properties of a given spatial relationship (Corcoran and Jones 2018), the most successful are the Intersection Model (Egenhofer 1991) and the Region Connection Calculus (Randell et al. 1992). Still, both these approaches and their generalizations (Schneider and Behr 2006) assume that the locations of connected objects, modeled as subsets of R^2, are accurately known. This is not always the case, because confounding factors may cause uncertainty in spatial data detection: among them, the most significant are incompleteness, due to lack of information, and vagueness, resulting from objects not having crisp or sharp boundaries (Worboys 1998). We aim to evaluate spatial objects whose geometries and shapes change over time: we will term these objects with the above-mentioned definition of moving regions (Liu and Schneider 2011) and will propose a model able to detect their conceivable topological modifications.

In technical terms, our goal was to achieve topological features to describe intersection or union of spatial moving disconnected regions in a two-dimensional Abelian lattice. At first, we built a two-dimensional grid made by 10×10 squares, each standing for a single pixel. The grid is composed of 100 white squares, representing empty locations. We randomly generated a pair of two-dimensional objects with different shapes, free to move inside the grid with unpredictable trajectories. The presence and movements of object shapes inside a video frame are pinpointed in terms their centers of mass (centroids). Over time, the shadows of shapes can overlap and sometimes merge, depending on the reflections (projections of the light blocked by the shapes). We benefited from the fact that a moving region whose location and extend change over time can imply topological changes such as hole formation and disappearance.

Hole detection was based on snapshots captured at instants termed observations. We formally defined a basic topological change, i.e., hole formation, between two consecutive observations (Amad and Peters 2018; Peters 2020). A hole is a bounded surface region that absorbs light. Shapes themselves are defined by their surface holes (e.g., spaces between an automobile grid, tire well, open windows). The detection method, modified from Gäuting et al. (2000), uniquely maps a unit, by partitioning the observations before and after the changes. A moving region can have different holes at different time intervals, and different changes between two same states may occur. For example, from a simple region to asingle region with holes, either a hole is formed inside the region (termed *hole form*), or the region touches itself at two ends (termed *region self-touch*).

The grid was equipped with sensors able to detect just a single feature: an empty square (a white pixel) fully surrounded by eight filled squares (red pixels). In other words, optical sensors, unable to detect object's shapes, occurrence and movements, are able to spot just the dynamical presence and disappearance of holes in the grid. Here follows the sensors' detection algorithm. At the Earth's near-ground level, holes can be found in surface object punctures and atmospheric water vapor puncture. From a video camera perspective, holes are regions that absorb light (Peters 2020). Both types of holes vary minutely in size and extent over time. To provide an example, atmospheric holes are continuously varying in location due to changing atmospheric pressure plasmas (Overzet et al. 2010). Variations in atmospheric holes also vary due to

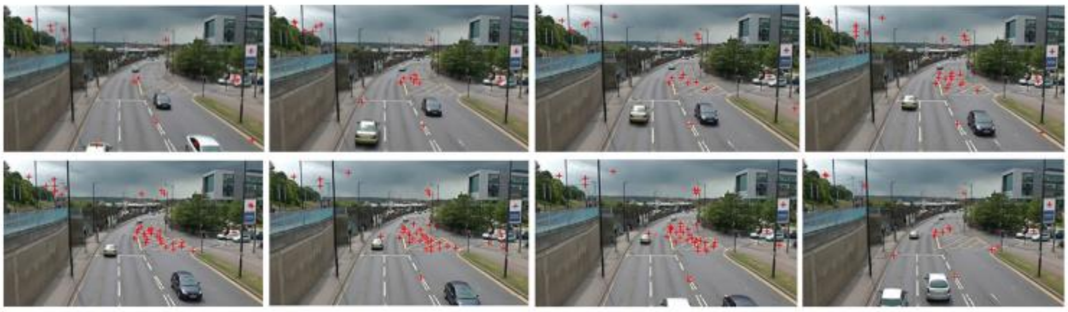

Fig. 1 Centroids on holes in a sequence of video frames. Each + (red plus sign) identifies the centroid of a hole. The locations of the holes in these video frames vary from moment due varying atmospheric conditions. (Color figure online)

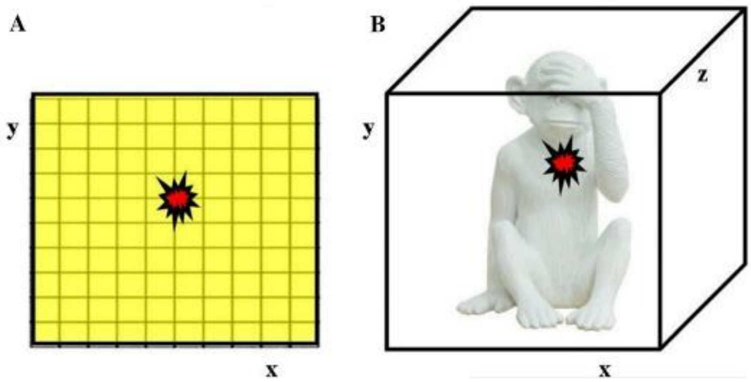

Fig. 2 Sample centroid holes, illustrated as red stars. **a** Centroid in a two-dimensional region at: (x, y) = (0.396, 0.554). **b** Centroid in a three-dimensional region at (x, y, z) = (0.0005, 0.0504,-0.7149)

movements of water molecules in the air and shifting sunlight conditions. Evidence of this can be seen in the sequence of video frames in the Fig. 1 below. Sample centroids (holes) of 2D and 3D regions are shown in Fig. 2.

The *centroid of a region* is the center of mass of the region. Let (x_i, y_j), $i = 1, \ldots, n$, $j = 1, \ldots, n$ be the coordinates of the points in an $n \times m$ region. And let (x_c, y_c) be the coordinates of the region centroid. Then the centroid of this 2D region is located at

$$(x_c, y_c) = \left(\frac{1}{n} \sum_{i=1}^{n} x_i, \frac{1}{n} \sum_{j=1}^{n} y_j \right)$$

Similarly, a centroid in a 3D region such as the regions in the video frames in the Fig. 1 is located at

$$(x_c, y_c, z_c) = \left(\frac{1}{m} \sum_{i=1}^{m} x_i, \frac{1}{m} \sum_{j=1}^{m} y_j, \frac{1}{m} \sum_{k=1}^{m} z_k \right)$$

The algorithm used to find the centroids of holes (dark regions) found in the video frames in the Fig. 1 is given next.

Input: Colour image **img** in a video frame.
Output: Centroids on holes recorded in a video frame.

Img → imgGrey % Convert img to greyscale.

imgGrey → imgBW % Convert imgGrey to binary.

find centroids of holes % Mark holes with +
End **Algorithm** Centroids of Video Frame Holes.

Results

Our technique allows the detection of holes in consecutive video frames that contain moving object shapes traveling in the 10 × 10 grid.

The Figure illustrates three on the countless possible distributions of the centroids of holes. At first, we assess the case of two intersecting moving shape shadows (Fig. 3a). Also, we consider the case which can be explained as shape reflection (projection the light blocked by a shape) resulting the formation of surface holes (Fig. 3b). Our account can be generalized to reflections of multiple moving object shapes whose dynamics include pinpointing the centroids of interior shape holes as well as reflected light leading to shadow holes (Fig. 3c). It is noteworthy that the knowledge of a single video frame parameter, i.e., the presence of pixels in the interior of the holes arising from the dynamical interaction of the moving shapes, allows us to achieve knowledge about topological features of otherwise undetectable objects. Indeed, even if an observer was unaware of the two moving objects' shapes and trajectories, that observer would be still able to extract useful information, simply looking at the detection screening that signals the presence (and disappearance) of holes. Although no information is available concerning the two objects' shape, nevertheless the observer is still able to detect and quantify their dynamics and interactions.

Conclusions

Concerning the biophysical realm, many accounts of our real world, both philosophical and biological, are used to consider a living being as an individual, self-preserving, unique subject equipped with borders, such as cellular membranes, envelopments, Markov blankets, and so on. The canonical illustration of biological organisms depicts "something" equipped with some peculiar activity, confined into itself, which struggles against (and cooperates with) the external world, in order to keep its entropy as low as possible and devoted to self-preservation (Ramstead et al. 2019). However, despite such stereotypical descriptions, the things get much more complicated when we approach the real word and its physical/biological content, in particular the brain function. The demarcation among objects, things and living beings could be somewhat arbitrary, and the same holds for set theory too. The same limitation holds for the very concept of evolution, which is based on species. Our concept of species is very arbitrary, because it throws not experimentally demonstrated borders among living beings. If we think to different populations of Homo, despite we are used to consider Sapiens, Neandertals, Denisovians, Floresians as different species, genetic studies undoubtedly point towards their ancient ibridation, which occurred more than once in different prehistoric contexts (Slon et al. 2018). Concerning our DNA, the delimitation of a species from another is sometimes difficult, due, e.g., to the presence in animal genome of viral

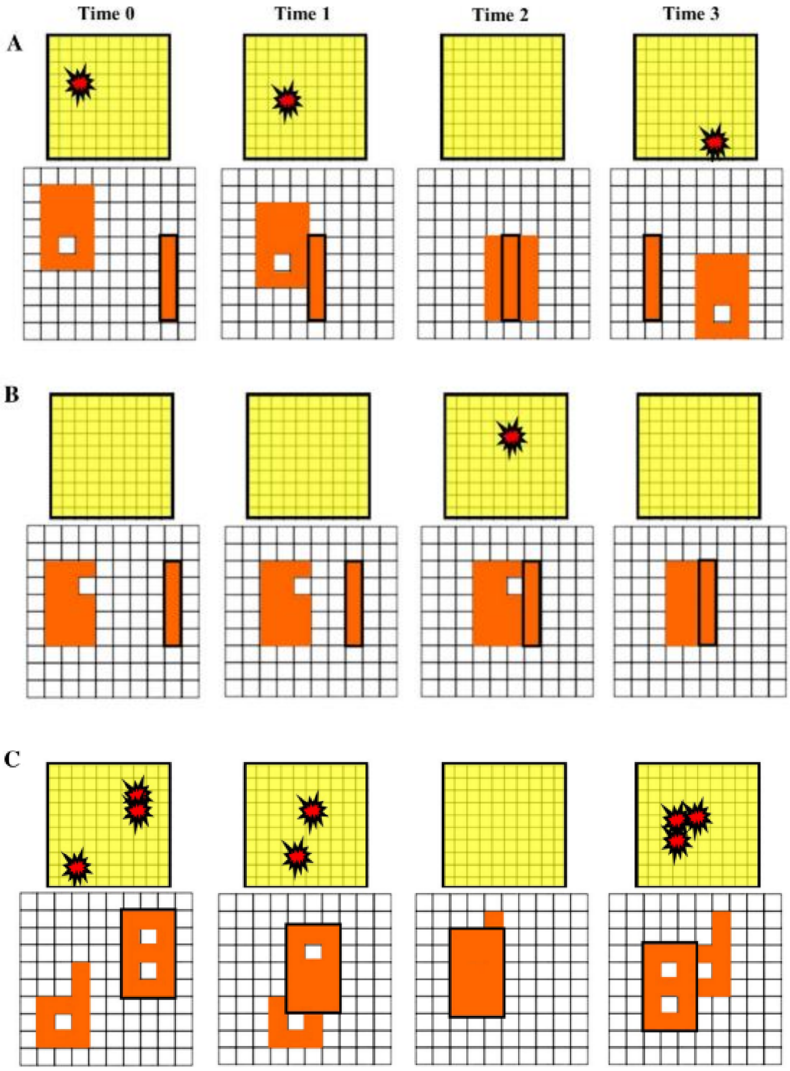

Fig. 3 Moving regions in a two-dimensional grid. The upper parts of each of the three figures (yellow grids) display the detection screening for hole recognition: when sensors detect holes, they fire inside the corresponding pixels (red stars). The lower part of the three figures (white grids encompassing red shapes) illustrates moving regions traveling inside a 10 × 10 grid. **a** Depicts the union of two objects, one with a hole and one without. **b** Displays a case of hole formation, when two shapes with ho holes interact and merge. **c** Depicts the dynamics of two moving shapes equipped with a different number of holes. (Color figure online)

and bacterial sequences. To make an example, the horizontal transmission of genetic material is able to overtake the so-called "species-specific" barriers (Chen et al. 2018), making sometimes difficult to clearly encompass individual living beings in a given species.

Given these premises, our aim here was to describe a technique to detect topological information from unknown and (possibly) disconnected spatial dynamic physical shape homeostasis features. We showed how unidentified interactions between two shapes on a two-dimensional grid give rise to modifications in topological features which can be detected and quantified. This means that topological changes pave the way to several possible applications in far-flung scientific disciplines. To provide a few examples, the detection of moving regions' holes might allow to assess the extent of a spreading forest fire, or the coverage of two mobile networking devices (Liu and Schneider 2011). The study of these topological changes might also help researchers to improve the performance of wireless sensor networks (Worboys and Duckham 2006) and the knowledge of neuronal homeostasis that counteracts long-term perturbations of neural activity and protein homeostasis in terms of is folding dynamics (Tozzi et al. 2016). Also, topological detection in network models might provide insights in both global structure and local node interactions, either in natural or artificial neural networks and in lattice percolation procedures. Further, unknown patterns assessed through topological weapons could be useful in understanding economic and sociologic trends of migratory fluxes (Ignacio and Darcy 2019). Our technique of hole detection makes it possible to assess the discrete dynamic changes and multi-dynamic clustering occurring in the brain as well as at the macroscales of galaxies, cosmic bodies and Megaparsec web-like cosmic matter distribution, describing the topological invariants of their subtending networks (Pranav et al. 2016) and, possibly, providing better insights in dark matter and vacuum energy. Our account might also provide unexpected insights in the physiological neural mechanisms of figure-ground perception, in scenes where near surfaces occlude multiple contours and borders at different depths, producing alignments that are improbable except under conditions of occlusion. Indeed, in touch with our account, Gillam and Grove (2011) hypothesized that figure-ground and hole perception can be determined solely by properties of ground, in the absence of any figural shape, or surround, factors. The last, but not the least, the foremost role of holes seems to corroborate the recently-introduced axiomatic foundations of the descriptive closeness of points and sets, which aims to overtake the conventional closeness of points and sets (Di Concilio et al. 2018). In the context of descriptive proximity, the closeness of points and sets can be viewed in terms of their overlapping descriptions of punctured surface homeostasis that delineate detectable holes with high acuity.

References

Alexandroff P (1961) Elementary concepts of topology. Dover Publications Inc., Ne w York

Amad MZ, Peters JF (2018) Maximal centroidal vortices in triangulations. A descriptive proximity framework in analyzing object shapes. Theory Appl Math Comput Sci 8(1):39–59

Arkhangelskii AV (2001) [1994]. Extremally-disconnected space. In: Hazewinkel M (eds) Encyclopedia of mathematics. Springer, Berlin. ISBN 978-1-55608-010-4

Autrecourt, Nicholas of. About 1340. The Universal Treatise. Marquette University Press, Milwaukee, Wisconsin (1971)

Chen J, Quiles-Puchalt N, Chiang YN, Bacigalupe R, Fillol-Salom A et al (2018) Genome hypermobility by lateral transduction. Science 362(6411):207–212. https://doi.org/10.1126/science.aat5867

Cooke GE, Finney RL (1967) Homology of cell complexes. Princeton University Press, Princeton

Corcoran P, Jones CB (2018) Stability and statistical inferences in the space of topological spatial relationships. IEEE Access 6:18907–18919. https://doi.org/10.1109/ACCESS.2018.2817493

Déli E, Tozzi A, Peters JF (2017) Relationships between short and fast brain timescales. Cognit Neurodyn 11(6):539–552

Di Concilio A, Guadagni C, Peters JF, Ramanna S (2018) Descriptive proximities. Properties and interplay between classical proximities and overlap. Math Comput Sci 12:91–106. https://doi.org/10.1007/s11786-017-0328-y

Edelsbrunner H, Morozov D (2012) Persistent homology: theory and practice. In: European Congress of Mathematics, Krakow, 2–7 July 2012, European Mathematical Society, pp 31–50

Edelsbrunner H, Letscher D, Zomorodian A (2002) Topological persistence and simplification. Discrete Comput Geom. https://doi.org/10.1007/s00454-002-2885-2

Egenhofer MJ (1991) Reasoning about binary topological relations. In: Symposium on spatial databases. Springer, pp 141–160

Fort C (1919). The book of damned. Prometheus Books, 1991

Güting RH, Böhlen MH, Erwig M, Lorentzos CSJNA, Schneider M, Vazirgiannis M (2000) A foundation for representing and querying moving objects. ACM Trans Database Syst 25(1):142

Ghrist R (2014) Elementary applied topology. Amazon. ISBN 978-1-5028-8085-1

Gillam BJ, Grove PM (2011) Contour entropy: a new determinant of perceiving ground or a hole. J Exp Psychol Human Percept Perform 37(3):750–757

Harnack D, Pelko M, Chaillet A, Chitour Y, van Rossum MCW (2015) Stability of neuronal networks with homeostatic regulation. PLoS Comput Biol. https://doi.org/10.1371/journal.pcbi.1004357

Ignacio PSP, Darcy IK (2019) Tracing patterns and shapes in remittance and migration networks via persistent homology. EPJ Data Sci 8:1. https://doi.org/10.1140/epjds/s13688-018-0179-z

Liu H, Schneider M (2011) Tracking continuous topological changes of complex moving regions. In: Proceeding SAC '11. Proceedings of the 2011 ACM symposium on applied computing. TaiChung, Taiwan. ACM, New York, NY, USA, pp 833–838. ISBN 978-1-4503-0113-8. https://doi.org/10.1145/1982185.1982366

Morimoto RI, Cuervo AM (2009) Protein homeostasis and aging: taking care of proteins from the cradle to the grave. J Gerontol A Biol Sci Med Sci 64A(2):167–170

Overzet LJ, Jung D, Mandra MA, Goeckner M, Dufour T, Dussart R, Lefaucheux P (2010) RF impedance measurements of DC atmospheric micro-discharges. Eur Phys J D 60:449–454. https://doi.org/10.1140/epjd/e2010-00274-5

Peters JF (2018a) Proximal planar shapes. Correspondence between triangulated shapes and nerve complexes. Bull Allahabad Math Soc 33:113–137

Peters JF (2018b) Proximal vortex cycles and vortex nerve structures. Non-concentric, nesting, possibly overlapping homology cell complexes. J Math Sci Model 1(2):56–72

Peters JF (2020) Computational geometry, topology and physics of digital images. Shape complexes, optical vortex nerves and proximities. Springer, Cham. https://doi.org/10.1007/978-3-030-22192-8

Popkin RH, Maia Neto JR (eds) (2007) Skepticism: an anthology. Prometheus Books. ISBN 1591024749; ISBN13 9781591024743

Pranav P, Edelsbrunner H, van de WeygaertR Vegter G, Kerber M et al (2016) The topology of the cosmic web in terms of persistent Betti numbers. Mon Not R Astron Soc 465(4):4281–4310. https://doi.org/10.1093/mnras/stw2862

Ramstead MJD, Constant A, Badcock PB, Friston KJ (2019) Variational ecology and the physics of sentient systems. Phys Life Rev 31:188–205. https://doi.org/10.1016/j.plrev.2018.12.002

Randell DA, Cui Z, Cohn AG (1992) A spatial logic based on regions and connection. In: Proceedings 3rd international conference on knowledge representation and reasoning

Rao AR (2018) An oscillatory neural network model that demonstrates the benefits of multisensory learning. Cognit Neurodyn 12(5):481–499

Schneider M, Behr T (2006) Topological relationships between complex spatial objects. ACM Trans Database Sys 31(1):39–81

Sen S, Daimi SN, Watanabe K, Takahashi K, Bhattacharya J, Saha G (2019) Switch or stay? Automatic classification of internal mental states in bistable perception. Cognit Neurodyn 14:95–113. https://doi.org/10.1007/s11571-019-09548-7

Slon V, Mafessoni F, Vernot B, de Filippo C, Grote S et al (2018) The genome of the offspring of a Neanderthal mother and a Denisovan father. Nature 561:113–116

Terasawa J (2004) Extremally disconnected spaces. In: Hart KP, Nagata J, Vaughan JE (eds) Encyclopedia of general topology. Elsevier, Amsterdam. https://doi.org/10.1016/b978-044450355-8/50092-1

Tozzi A, Peters JF (2017) From abstract topology to real thermodynamic brain activity. Cognit Neurodyn 11(3):283–292

Tozzi A, Peters JF (2019) Points and lines inside human brains. Cognit Neurodyn 13(5):417–428

Tozzi A, Tor Fla, Peters JF (2016) Building a minimum frustration framework for brain functions in long timescales. J Neurosci Res 94(8):702–716

Verri A, Uras C (1997) Metric-topological approach to shape representation and recognition. Image Vis Comput 14:189–207

Verri A, Uras C, Frosini P, Ferri M (1993) On the use of size functions for shape analysis. Biol Cybern 70:99–107

Wang P, Hu J (2019) A hybrid model for EEG-based gender recognition. Cognit Neurodyn 13(6):541–554

Worboys M (1998) Imprecision in finite resolution spatial data. GeoInformatica 2(3):257–279

Worboys M, Duckham M (2006) Monitoring qualitative spatiotemporal change for geosensor networks. Int J Geogr Inf Sci 20:1087–1108

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.