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# Cellular Gauge Symmetry and the Li Organization Principle: A Mathematical Addendum. Quantifying energetic dynamics in physical and biological systems through a simple geometric tool and geodetic curves

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## ABSTRACT

The present Addendum complements the accompanying paper “Cellular Gauge Symmetry and the Li Organization Principle”; it illustrates a recently-developed geometrical physical model able to assess electronic movements and energetic paths in atomic shells. The model describes a multi-level system of circular, wavy and zigzag paths which can be projected onto a horizontal tape. This model ushers in a visual interpretation of the distribution of atomic electrons' energy levels and the corresponding quantum numbers through rather simple tools, such as compasses, rulers and straightforward calculations. Here we show how this geometrical model, with the due corrections, among them the use of geodetic curves, might be able to describe and quantify the structure and the temporal development of countless physical and biological systems, from Langevin equations for random paths, to symmetry breaks occurring ubiquitously in physical and biological phenomena, to the relationships among different frequencies of EEG electric spikes. Therefore, in our work we explore the possible association of binomial distribution and geodetic curves configuring a uniform approach for the research of natural phenomena, in biology, medicine or the neurosciences.

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## 1. A new geometric model

Visual geometric models are widely used for the description of different phenomena in nature. In particular, the conception of a geodetic line or a geodetic curve was first considered long ago by Gauss (Bühler, 1981). To make a recent example, Yurkin (2017) provided a novel geometrical model of a structure of atomic electron shells, able to assess the main, azimuthal, magnetic, and spin numbers in terms of simple wavy trajectories, angles and distances. These parameters can be mapped onto a horizontal tape, making it possible us to assess difficult system structures by using an effortless MS Excel program, without additional proofs and assumptions.

Here follows a brief description of Yurkin's original geometric model for atomic shells (Yurkin, 2015). The geometrical steps in order to build atomic shells and subshells in the form of wavy trajectories (extended along horizontal tapes and connected with the image of circular atomic orbits) are summarized in Fig. 1 and Fig. 2. The paths are expressed in terms of wavy and circular trajectories of electron movements. Ratios between atomic models' features, as well as their quantum numbers and power levels, can be interpreted as trajectories of different shapes. In technical terms, Yurkin described a paraxial binomial (sharing in two) flat system of trajectories (Yurkin, 2016). This system consists of groups of rays, inclined under $p$ angles that are a) are smaller than a given axis, and b) multiple to an angle $\gamma$:

$$p = \left(\frac{m+1}{2}\right)\gamma,$$

where

$$m = 0,\ \pm1,\ \pm2\ldots.$$

In addition, let us accept that:

$$p_0 = \left(\pm\frac{1}{2}\right)\gamma = s\gamma,$$

where

$$s = \pm\frac{1}{2}$$

in case of $m = 0$.

Points of ray branching are spaced by a symmetry axis on small distances of a $q$ multiple to $k/2$ length:

$$q = \frac{jk}{2},$$

where $j = 0,\ \pm1,\ \pm2\ldots.$

We termed this rays system: “$[p = \left(\frac{m+1}{2}\right)\gamma, q = \frac{jk}{2}] -$ system”, or *half-integer ray system*. This binomial system consists of groups of rays $\mathbb{N}$ equipped with links $\mathbb{K}$ (Yurkin, 2016).

We assume that the rays display an energy $\boldsymbol{E}$, extending along the branching $\mathbb{K}$ links. Therefore, the number of rays of $\mathbb{N}$ (and their energy $\boldsymbol{E}$) can be easily summarized. Generally, $\mathbb{N} \geq \mathbb{K}$. We designate number of passes of rays along links as. $0,\ 1,\ 2\ldots,\ \mathrm{n}-1,\ \mathrm{n},\ \mathrm{n}+1\ldots$

This ray system can be placed in a rectangular coordinate grid. The size of a cell grid displays height $= \frac{k}{2}$ and length $= L$. The configurations of this two-dimensional system matches the real electronic distribution in various atomic shells (Yurkin, 2016), therefore it demonstrated to be a very accurate model. In the present paper, we generalize the proposed geometric model by using geodetic curves.

### 1.1. Geometrical counterparts of quantum systems' energy levels

Consider the following designations. Let $\boldsymbol{E}(\boldsymbol{n},\boldsymbol{l})'$ denote the energy extending along the ray characterized by numbers $\boldsymbol{n}$ and $\boldsymbol{l}$ at each odd pass of n, and let $\boldsymbol{E}(\boldsymbol{n},\boldsymbol{l})''$ denote the energy extending along the ray characterized by numbers $\boldsymbol{n}$ and $\boldsymbol{l}$ at each even pass of $\mathrm{n}-1$ or $\mathrm{n}+1$. Here $\mathrm{n}-1$, n and $\mathrm{n}+1$ are numbers of rays passes. The numbers $\boldsymbol{n}$ and $\boldsymbol{l}$ characterize each one of the rays of our binomial system.

Let the average energy between two neighbor passes be represented by

$$\boldsymbol{E}(\boldsymbol{n},\boldsymbol{l}) = \frac{\left[\boldsymbol{E}(\boldsymbol{n},\boldsymbol{l})' + \boldsymbol{E}(\boldsymbol{n},\boldsymbol{l})'^{\prime}\right]}{2}$$

Generally,$\boldsymbol{n}$ and $\boldsymbol{l}$ accept the following values:

$$\boldsymbol{n} = 1, 2, 3, \ldots$$

$$\boldsymbol{l} = 0, 1, 2, \ldots.$$

### 1.2. Quantifying angles, quantum numbers and ratios

In quantum mechanics, every electron is characterized by four physical quantum numbers. Yurkin's atomic model, in touch with the literature (2017), utilizes the following values:

Principal: $\boldsymbol{n}$ corresponds to. $\boldsymbol{n} = 1, 2, 3, \ldots$

Azimuthal: $\boldsymbol{l}$ corresponds to. $\boldsymbol{l} = 0, 1, 2, \ldots$

Magnetic: $\boldsymbol{m_l}$ corresponds to. $m = 0, \pm1, \pm2\ldots$

Spin: $\boldsymbol{m_s}$ corresponds to. $s = \pm\frac{1}{2}$

Note that first expression:

$$p = \left(m + \frac{1}{2}\right)\gamma,$$

can be written also as:

$$p = \left(m + \frac{1}{2}\right)\gamma = \pm\left(l + \frac{1}{2}\right)\gamma.$$

If we accept that the quantum system contains a finite number of electrons, then the above mentioned expressions of the four physical quantum numbers can accept the minimum and maximum values$\boldsymbol{n}(min), \boldsymbol{n}(max), \boldsymbol{l}(max)$, and so on.

## 2. Possible applications of the model in biophysics, physiology (human disease), and neuroscience

In the previous Section, in touch with Yurkin (2015, 2016, 2017), we illustrated how to build a geometrical grid that makes possible us to describe systems equipped with various energetic levels. Visual geometric models are widely used for the description of different natural phenomena (Peters, 2016, 2017a, 2017b; Frankel, 2016). To make an example, the periodic table of chemical elements displays periodic regularities that can be visually illustrated in the form of circles, pyramids, spirals, and so on. In point of fact, Yurkin's model is different from others: although it has been developed in order to assess electrons' energetic levels in atomic orbits, it might be extended, with the due corrections, to countless, quite different physical and biological systems. In Yurkin's atomic model, the four sought-after values, i.e., m and s (that describe the

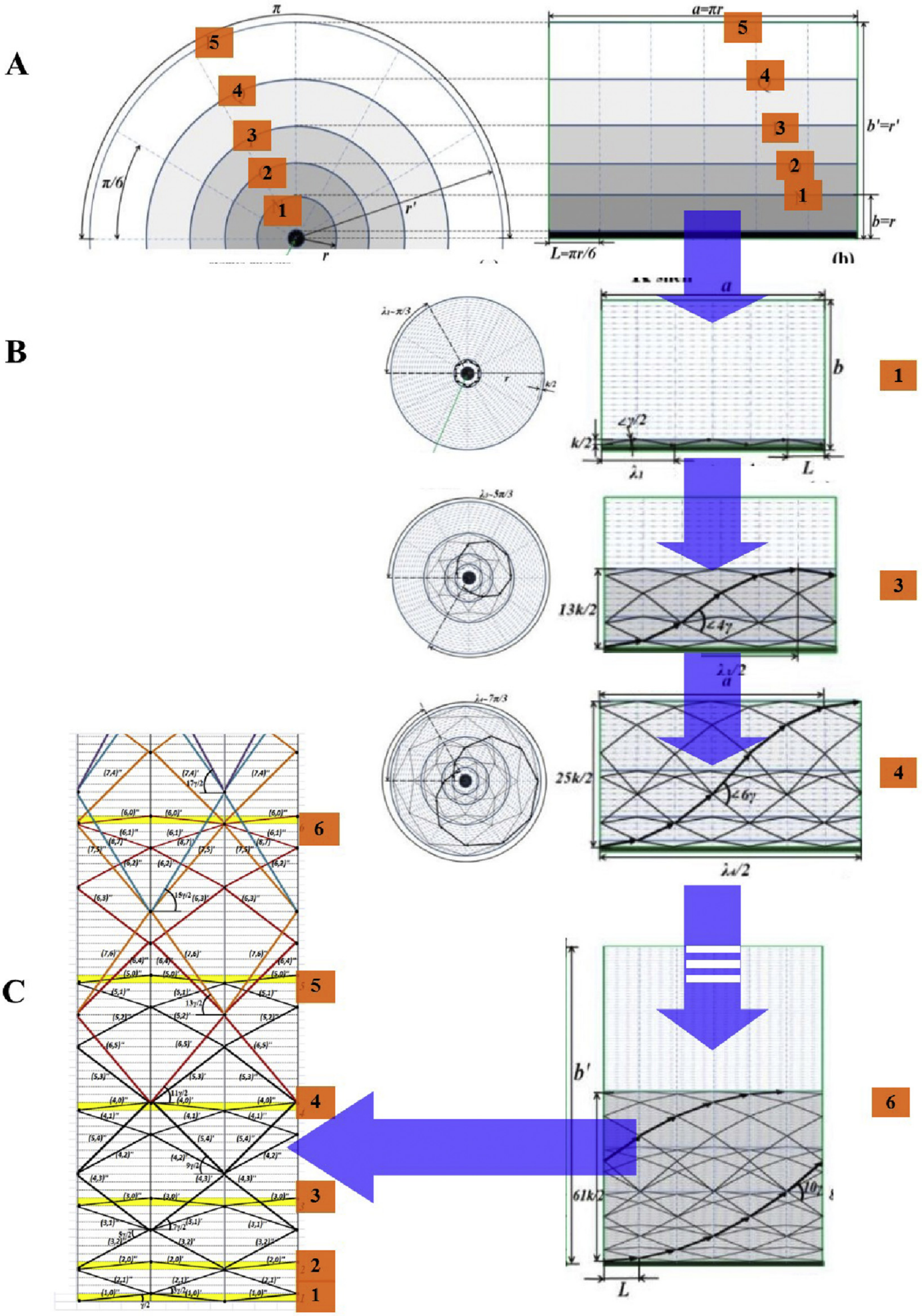

**Fig. 1.** Geometric model of atomic shells. Angles and energetic levels are calculated based on the movements of electrons in different orbits. Fig. 1A, **left**: five energetic levels are displayed as rotations on a circle: the lowest energy, close to the circle center, is numbered with 1, the highest with 5. Fig. 1A, **right**: the five circular orbits can be mapped onto a horizontal tape. Fig. 1B: the electronic movements on every energetic level can be depicted as wavy trajectories extended along the horizontal tape. Trajectories intersect, forming angles γ that vary according to ray inclination. The right and the left parts of Fig. 1B display views from different angulations. Fig. 1C, **right side**, displays the trajectories of a further, not shown level, the sixth one. Fig. 1C, **left side** illustrates a 2D geometrical picture of the 6-level atomic system, standing for the first six completely filled atomic shells. (Yurkin, 2017; modified).

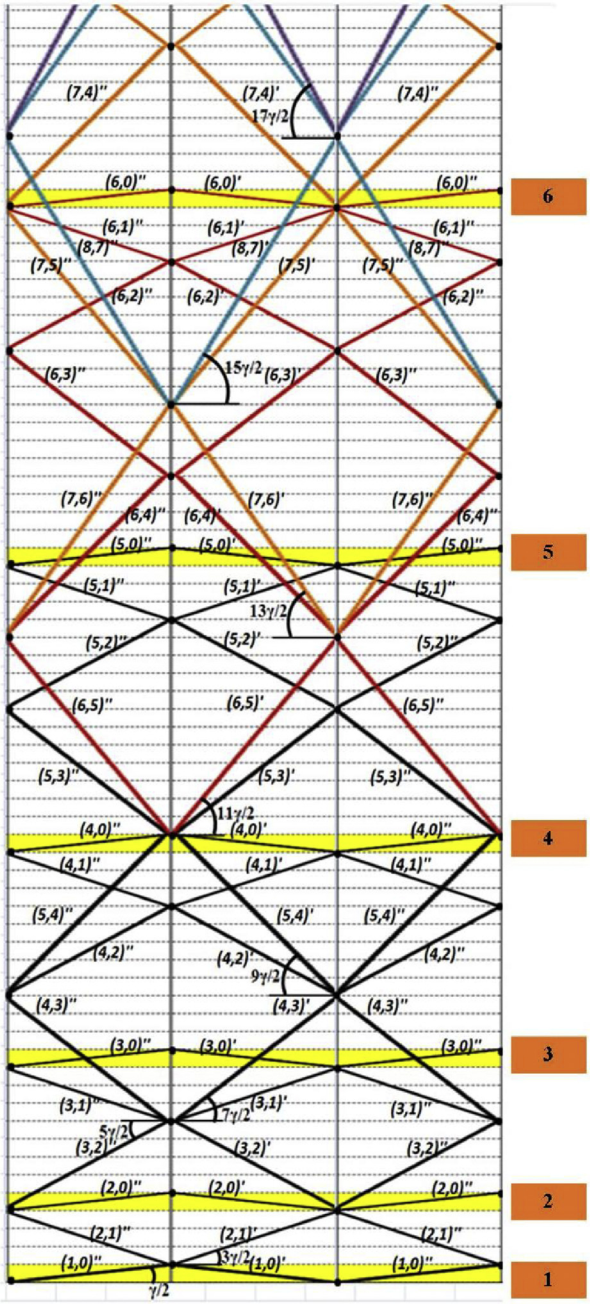

**Fig. 2.** Images of wavy trajectories of the first six completely filled atom shells and an of multilevel quantum system. To make an example, the first shell, termed with the squared number 1, consists of links inclined on angles γ/2 to an horizontal axis. The length of every line connecting two nodes stands for the average energy within two neighbor passes. Indeed, the energy $\boldsymbol{E}(\boldsymbol{n},\boldsymbol{l})^{'}$ and $\boldsymbol{E}(\boldsymbol{n},\boldsymbol{l})^{''}$ extending along these links is characterized by the numbers$\boldsymbol{n} = 1$ and $\boldsymbol{l} = 0$. The second shell, termed with the squared number 2, consists of links inclined on angles $\frac{\gamma}{2}$ and $\frac{3\gamma}{2}$ to a horizontal, while the energy $\boldsymbol{E}(\boldsymbol{n},\boldsymbol{l})'$ and $\boldsymbol{E}(\boldsymbol{n},\boldsymbol{l})^{''}$ is characterized by the numbers $\boldsymbol{n} = 1,2$ and $\boldsymbol{l} = 0,1$.

required angles) and n and l (that describe the required energetic levels), stand for four physical quantum numbers such as systems of optical solitons. Nevertheless, the four parameters might stand for other systems' features too, according to the specific requirements of the phenomenon under evaluation. In the accompanying paper, we already raised the possibility to describe cellular metabolic paths in terms of angles and polygonal sides endowed in the Yurkin's geometric grid. In particular, we provided the example of second messengers (cAMP) and their correlated energetic pathways. Here we will make available other examples of physical and biological systems that can be described in terms of the geometrical Yurkin's grid. Furthermore, we will discuss its invaluable advantages in terms of the possibility to assess hidden inter-level system's correlations.

### 2.1. *Timescales of protein folding*

Our geometrical model is able to assess not just static “spatial” systems equipped with intertwined multi-levels of energy, but also the “temporal” energetic and entropic evolution of natural phenomena. In such a vein, every process described by Fokker-Planck equations, e.g., random processes that dissipate energy, may be located in a geometric grid which allows their easier assessment. The possibility to compare various angles and paths on the same grid allows us to evaluate the presence of unexpected correlations among features lying on different systems' energetic levels. In protein folding, for instance, if we equip every folding phase with the proper geometrical level, we could be able, by evaluating the current state in terms of angles and rays, to achieve information about the previous and future states.

The key to understanding the concepts of free-energy, entropy and funnelling in the mechanisms of protein folding lies in the well-known suggestion that an energetic gap occurs in their fitness landscape, between a well-defined global minimum state and a dynamic transitional one, which simulates the system's behaviour before it settles into the global minimum, *i.e.*, fixed-point attractors endowed by the landscape. Amidst the valleys and peaks, larger variations in energy gap lead to huge, deep funnels, which inexorably draw the protein folding activities to themselves, in long timescales (Tozzi et al., 2016a,b). The different timescales occurring in protein folding processes give rise to constrained functional outcomes, conditioned only by long time-scale statistics. Thus, the importance of topology in a protein context lies in the different timescales. Fig. 3 depicts the process of protein folding in terms of energy landscape theory, translated to our geometrical grids. The required values of *m* and *s* and *n* and l might stand, in this context, for the parameters that give rise to protein folding, depicted on the *x*, *y*, and *z* axes of Fig. 3A.

### 2.2. *The physiological underpinning of human disease*

The geometrical grid, when applied to biochemical paths occurring in our cellular assembly, or to whole body images, of to histological pictures of human normal and pathological samples, allows us to predict the real values in every point of the grid. A perturbation, *e.g.*, a disease, occurring in one of the points can be geometrically evaluated in terms of grid deformations that lead to local and global changes is angles and paths. The knowledge of the “normal”, physiological geometrical grid of metabolic paths allows us to understand and predict where deformations might occur during pathological events. To make an example, if we detect a local grid deformation according with the shape of the geodetic curves, such as, e.g., a metastasis of unknown source, we could be able, by assessing the changes occurred in other parts of the grid, to understand where the primary source of the tumor is located (see Fig. 4). In other words, if we accurately know the angles and paths of the normal grid, we could be able to find hidden injuries, by just looking at a secondary alteration in another place of the grid.

### 2.3. *Assessing brain oscillations*

Brain electric activity displays a mean action potential rate of 4 Hz (Sengupta et al., 2013a,b), which is just a mean value

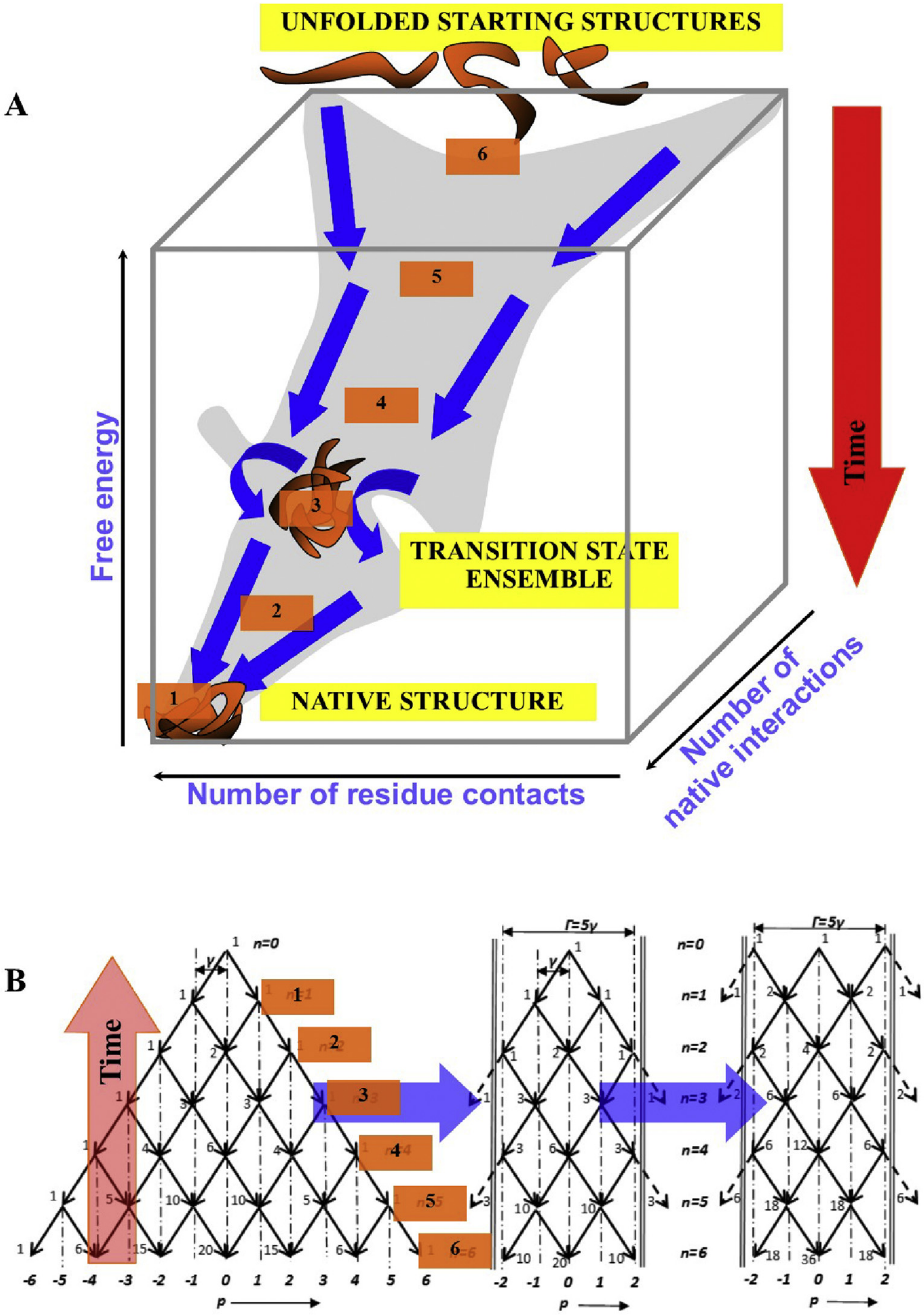

**Fig. 3.** Protein folding framed in the context o the energy landscape theory. Fig. 3A represents the folding sequence as equipped with a funnelling mechanism dictated by long timescales. (Modified from Tozzi et al., 2016a,b). Fig. 3B depicts regular, non-chaotic branching trajectories (of the first binomial type) of the energetic landscape depicted in Fig. 3A inside a neuronal network.

accounting for the diverse timescales of nervous activity. The coexistence of very low-frequencies (<2 Hz) and high frequencies (>10) is well established in neuroscientific studies (Buzsaki and Watson, 2012). It has been proposed that the different timescales are associated with unrelated functional activities (Raichle, 2010; de Arcangelis and Herrmann, 2010; de Pasquale and Marzetti, 2014). To make an example, it has been argued that the: a) anatomical, b) functional and c) psychological correlates of the very slow cortical potentials are respectively: a) the spontaneous brain activity (Raichle, 2010), b) the default mode networks and c) the unconstrained, conscious cognition (i.e. mind-wandering or day dreaming propensities) (Andrews-Hanna et al., 2014; Kucyi and Davis, 2014) or the dreaming state (Fox et al., 2015). It has been suggested that spontaneous low-frequency cerebral fluctuations are a neuronal activity that is intrinsically generated by the brain (Fox and Raichle, 2007). On the other hand, during stimuli-evoked activity or cognitive demands, the brain transitorily exhibits other functional conformations, linked with specific psychological correlates (Cole et al., 2014). For example, the brain high frequency activity has been associated with perceptual binding and feedback/

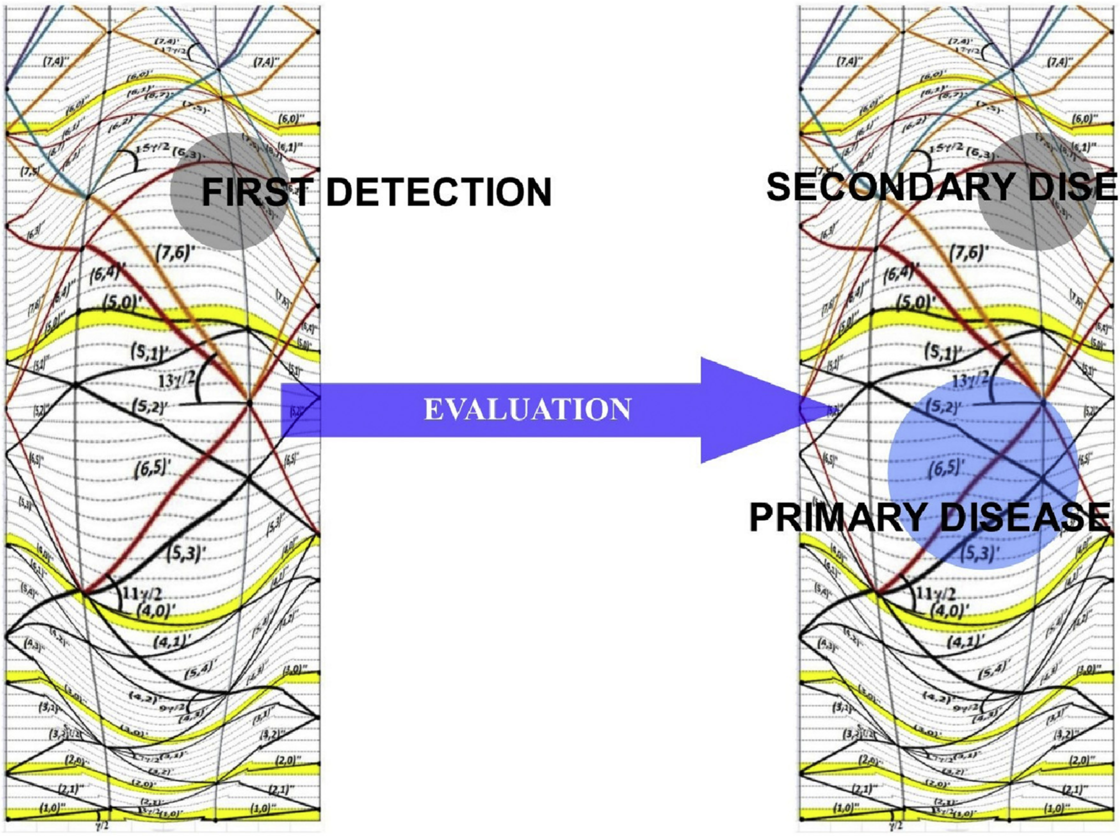

**Fig. 4.** Illustrating wavy trajectories of a multilevel cellular pathway, equipped with six increasing energetic levels. As always, the lower energetic level is placed at the bottom, the higher at the top. When an external force acts on the system (**left side**), a grid deformation according with the shape of the geodetic curves occurs, so that the angles γ and the energy lines are modified. The system's broken homeostasis, e.g., the grid deformation, is transmitted to other areas. The right part of the Figure shows how it is possible, starting from the initial detected perturbation, to derivate the primitive deformation.

feedforward waves able to improve the perception of external inputs (Buzsaki and Watson, 2012).

Given that the (environmental and internal) inputs provoke changes in spike frequency in diverse cortical areas, the brain exhibits a high number of possible source configurations, such as gamma oscillations in somatomotor cortex during states of enhanced vigilance, or alpha waves in posterior zones with eyes open, and so on (Bastos et al., 2014). Here we ask whether our geometrical model is able to detect hypothetical correlations between the functionally heterogeneous frequencies, and evaluate whether it is true that, when one changes, also the other modifies. Can the vast repertoire of brain electric spikes with different frequencies be evaluated in geometrical terms?

In order to build the proper grid, we need to start from the brain's energy requirements in different functional states. Evoked activity (i.e. perceptual and motor activity, task performance and similar cognitive functions), requires an additional energy consumption of ~5%, as compared with spontaneous activity. This is due to the local increases in spike frequency (in particular beta and gamma waves) that occur during evoked activity. The increased spike frequency during evoked activity has a significant metabolic cost, specifically 6.5 μmol/ATP/gr/min for each additional spike, with an ATP consumption higher than the “typical” mean rate at 4 Hz (which is $3.29 \times 10^9$ molecules of ATP/neuron/sec) (Attwell and Laughlin, 2001; Sengupta et al., 2013a,b). This led Tozzi et al. (2016a,b) to argue that the spontaneous brain activity consumes an amount of energy lower than the evoked activity. Indeed, each spike is equipped with an oscillation, characterized by two parameters: amplitude and frequency. For the Ohm's law, energy consumption due to the AMPLITUDE of the oscillation is negligible as compared with energy consumption due to the FREQUENCY of the oscillation. That means that evoked activity, equipped with a mean frequency of spikes higher than spontaneous activity, expends more energy. The higher the spike frequency, the higher the energetic level. This statement allows us to analyze brain spike frequencies in terms of a Yurkin's multilevel grid, where the lowest frequencies (<2) stand for the level 1 and the gamma spikes stand for the highest level (that in Fig. 5, is the level 6).

The amount of energy E can be calculated through the above mentioned ATP consumption for every spike. Other required parameters, such as m and s and n and l, might stand for different possible brain features, such as, to make an example, the spike orientation in the brain (feed-forward or feed-back), the trains of synchronized spikes of the same frequency, phase opposition, hierarchical or not hierarchical features, the interaction strength among the single neural nodes, and so on and so on (Fig. 6).

When encompassed in the proper geometrical grid, the different brain oscillations clearly exhibit correlations and energetic relationships, illustrated by angles' and side lengths'inter-levels matching descriptions. Indeed, a geometrical view of the brain spikes allows us to assess different frequencies in algebraic terms of homotopies and affine connections. Embracing brain spikes in the framework of geometry and topology (Dodson and Parker, 1997; Tozzi and Peters, 2016) means that biological brain timescales can be described as functions on grids: this perspective permits the nervous oscillations located in the “real” space (the 3D brain) to be

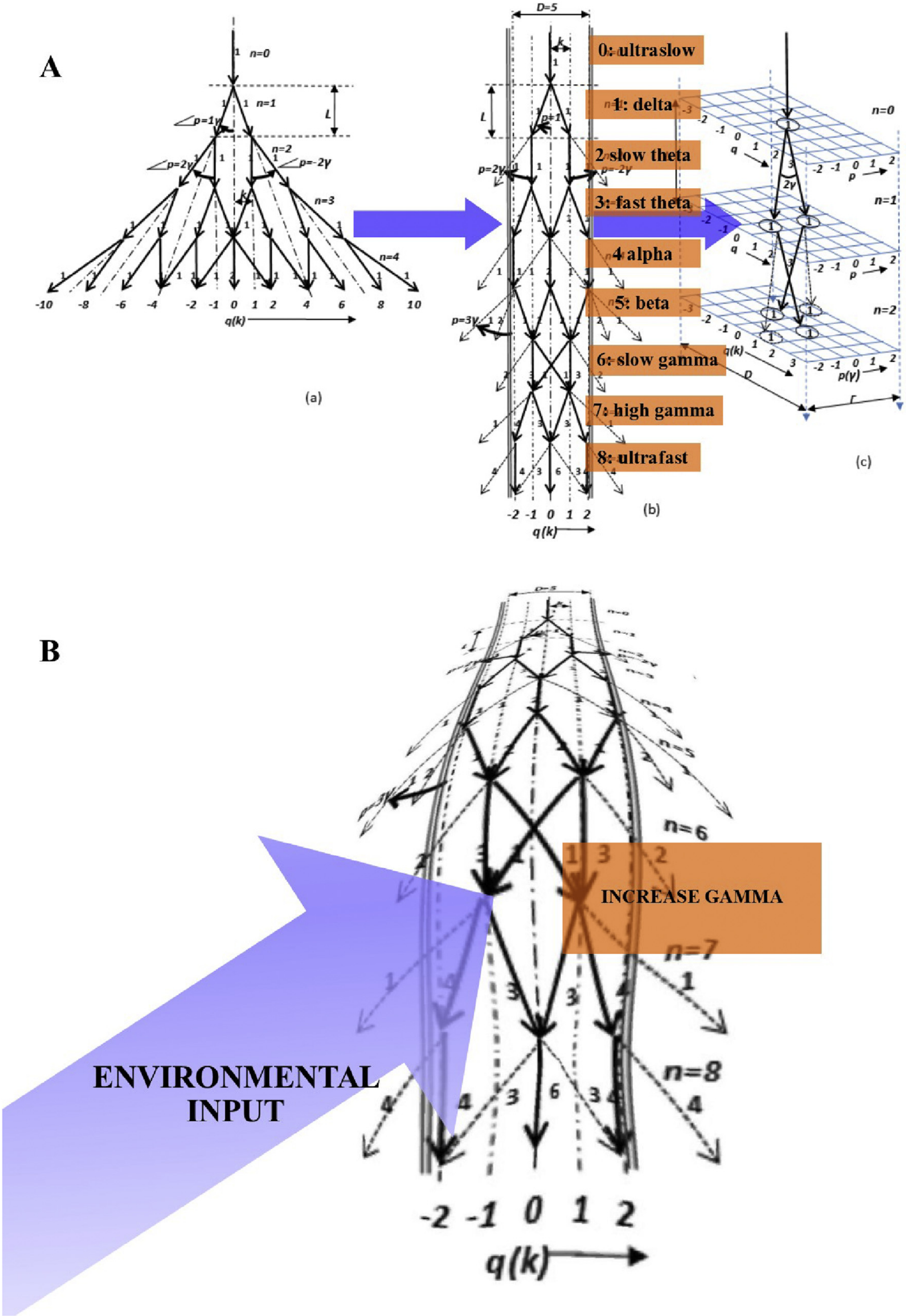

**Fig. 5.** Regular, non-chaotic and branching trajectories inside a neuronal network. The Figure displays neural trajectories of the second binomial type. Each neuron could connect with one, or two, or many other neurons. The hypothetical Fig. 5B depict what happens when an environmental input activates the higher brain frequencies. A grid deformation according with the shape of the geodetic curves takes place, starting from the higher frequencies, and extending to the lower frequencies in a predictable, testable and assessable way.

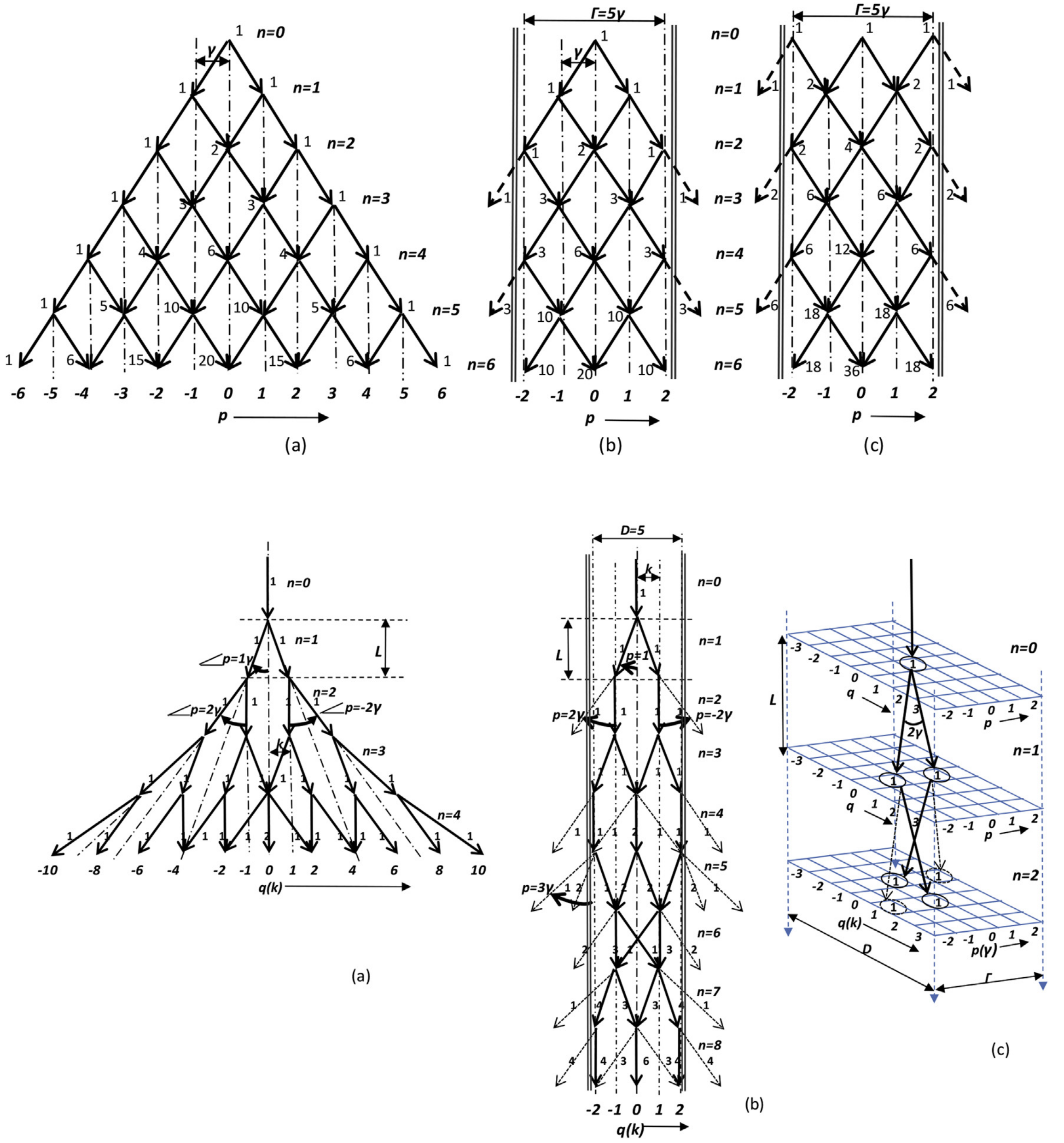

**Fig. 6.** Not chaotic, regular branching trajectories inside a neuronal network. **The upper part** illustrates neural trajectories of the first binomial type. **The lower part** displays neural trajectories of the second binomial type. Each neuron could connect with one, or two, or many other neurons.

translated to “abstract” spaces and *vice-versa,* enabling us to achieve maps from one level to another that allow the comparison of otherwise hidden features.

One of the main concerns in neuroscience is to try to understand how brain oscillations, frequencies and amplitudes are correlated. By inserting every range of spike frequencies in a different level of our geometric grid, we could be able to detect their hidden correlations, described by matching angles and the same paths lengths. To make a practical example, we could be able to study what happens to frequencies when just one of them is activated (Fig. 5B). It is well-established that perception is linked with trains of gamma and beta frequencies. In turn, gamma and beta frequencies, in our framework, stand for deformations in the higher energetic levels in the grid. In this case, for a more accurate description of real processes, we use geodetic curves, rather than straight lines.

Because we know the geometric relationships among the different spike frequencies at rest, we may be able to understand how the lower grid levels (i.e., the lower brain frequencies) respond to deformations in the higher levels (i.e., the task-related activation of the higher brain frequencies).

## 3. Conclusion

The ideal point of rest, about which the life of the organism moves in constant oscillations, has also a mathematical-logical significance when framed as a geometrical theory. Yurkin's approach allows us the possibility to compare various angles and paths on the same grid to evaluate the presence of unexpected correlations among features lying on different systems' energetic levels. If we equip every system with the proper geometrical level, we might be able, by evaluating the current state in terms of angles and rays, to achieve information about the previous and future states. In particular, Yurkin's approach shows that spikes of different frequencies are correlated in the brain, because they display regular, comparable features and relationships. By knowing the values of a few grid points, the other ones can be detected and measured on other zones of the grid, because geometry provides a feasible way to link their reciprocal activities. Therefore, a theory from an apparent far-flung branch, the geometry, might help us to improve our knowledge of biomolecular and physiological phenomena, as well as the different nervous oscillations' relationships.

It could be asked whether a specific geometry of biological systems is feasible, and whether is it possible to cogently describe the cell based on very few first principles as geometry does. In this paper, we try to do that, and we even go further. By paraphrasing the influential paper from Wigner (1960), we highlight the "unreasonable power of mathematics in biology". Indeed, mathematical geometrical concepts turn up in entirely unexpected connections in the study of cellular function: they permit a surprisingly close and accurate description of biological phenomena in these connections. Indeed, in spite of the baffling complexity of the cell, geometry allows the discovery and the description of certain biological regularities and astonishing invariants properties that can be formulated in terms of accurate mathematical concepts.

We have showed in the accompanying paper and in this Addendum how gauge theories and geometry may play a sovereign role in the cellular affairs, so that the laws of a homeostatic living being can also be formulated directly in the language of mathematics, which seems to be the most universal language of science. Life does not abhor mathematical description, but demands the exploration of new biomathematical conceptualizations more congenial to biological complexity than the formal tools developed in the last two centuries. In the present work, starting from a top-down mathematical approach, we may be able to assess the apparently irreconcilable physical and biological coins of the cell without any reductionism. In sum, paraphrasing Wigner once again, we do not know why geometry works so well, but its accuracy seems to override the concerns about its truth and consistency.

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